Combinatorics 2017 HW 1009

Student ID: **2018280070** Name: **Peter Garamvoelgyi**  Score:

**1. how many integers from 1 to 10000 are not squares of integers or cubes of integers？**

First, let us define some sets:

* Let be the universal set.
* Let the set of squares within.
* Let the set of cubes within.

Our goal is to find . Using the *inclusion-exclusion principle*:

The cardinality of these sets:

(For to be both a square and a cube,has to be divisible by both 2 and 3, i.e. .)

Substituting these values, we get

**There are 9883 integers from 1 to 10000 that are neither squares nor cubes of integers.**

**2. How many permutations of 1, 2, 3，……，9 have at least one odd number in its natural position?**

Let the property denote that the number is in its natural position. Let denote the set of permutations satisfying. Our goal is to find the number of permutations satisfying at least one of the properties

To satisfy , the number has to be in its natural position, and the rest of the numbers can be arranged in any way:

For satisfying two properties and , the numbers and have to be in their natural positions, while the rest can be arranged in any way:

The same method can be extended to more constraints. For all properties, we get:

Having defined the cardinality of these sets, we can now calculate our final value using *IEP*:

**Thus, there are 157,824 permutations of the numbers 1, 2, …, 9 that have at least one odd number in its natural position.**

**3.**

**please calculate the number of integral solutions.**

Let us reformulate the equation:

Let denote the property that the constraint for is violated. Let denote the set of integral solutions with property. Our target is

When counting the number of solutions violating, we can replace with whereis the upper constraint associated withand. The solutions of the resulting equations (without taking any upper bound into account) are exactly the solutions of the original equation violating.

From this, we can get the final result using IEP:

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**There are 96 integral solutions to the given equation with the given constraints.**

**4. For the permutation P=P1 P2 P3 P4 of {1,2,3,4}, how many feasible permutations are there if we constrain that P1≠2，P2≠2、3， P3≠3、4， P4≠4? (4 points)**

Let denote the set of permutations violating the constraint for. Our goal is to find

Let us count the possibilities:

From this, we can get the final result using *IEP*:

**… i.e. there are 4 permutations that satisfy the given constraints.**

Using the permutation generator from project 2, we can check the result:

bool check\_constraints(const std::vector<int>& vec) {

return (vec[0] != 2) && (vec[1] != 2 && vec[1] != 3) &&

(vec[2] != 3 && vec[2] != 4) && (vec[3] != 4);

}

int main() {

Lexicographic<int> g = {1, 2, 3, 4};

do {

if (check\_constraints(g.get())) print\_vector(g.get());

} while (g.next());

}

The output is the 4 permutations: 1 4 2 3, 3 4 1 2, 3 4 2 1, 4 1 2 3.